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TEACHING MATERIAL ON

MATHEMATICS

SCHOOL OF SCIENCE

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Two-Phase Method

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(1) Use two-phase simplex method to solve the following LP problem.

Minimize $z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$

Sol^n:→ Standard form

$Max\ (Z) = -x_1 - x_2$

Subject to $-2x_1 - x_2 \leq -4$

$-x_1 - 7x_2 \leq -7$

and $x_1, x_2 \geq 0$

After adding Surplus variables

$Max\ (Z) = -x_1 - x_2 + 0S_1 + 0S_2$

Subject to $-2x_1 - x_2 + S_1 = -4$

$-x_1 - 7x_2 + S_2 = -7$

and $x_1, x_2 \geq 0$

Sol^n:- After Adding Surplus variables

$Max(Z) = -x_1 - x_2$

subject to $2x_1 + x_2 - S_1 + A_1 = 4$

$x_1 + 7x_2 - S_2 + A_2 = 7$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Phase – I

→ Auxiliary Linear Programming problem

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$$\text{Max } z^* = +0x_1 + 0x_2 + 0S_1 + 0S_2 - A_1 - A_2$$

Subject to $2x_1 + x_2 - S_1 + A_1 = 4$

$x_1 + 7x_2 - S_2 + A_2 = 7$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

		c_j	0	0	0	0	-1	-1	
co-eff^n of Basic variables	variables in Basis	sol^n value	x_1	x_2	S_1	S_2	A_1	A_2	Min^n ratio
-1	A_1	4	2	1	-1	0	1	0	4
-1	A_2	7	1	7	0	-1	0	1	1 →
$z^* = -11$		z_j	-3	-8	1	1	-1	-1	
		$c_j - z_j$	3	8 ↑	-1	-1	0	0	

~~$R_2 \to \frac{1}{7}$~~ $R_2(\text{New}) \to \frac{1}{7} R_2(\text{Old})$

$R_1(\text{New}) \to R_1(\text{Old}) - R_2(\text{New})$

		c_j	0	0	0	0	-1	
co-eff^n of Basic variables	variables in Basis	~~Value~~ sol^n Value	x_1	x_2	S_1	S_2	A_1	Min^n ratio
-1	A_1	3	$13/7$	0	-1	$1/7$	1	$21/13$ →
0	x_2	1	$1/7$	1	0	$-1/7$	0	7
$z^* = -3$		z_j	$-13/4$	0	1	$-1/7$	-1	
		$c_j - z_j$	$13/4$ ↑	0	-1	$1/7$	0	

$R_1(\text{New}) \rightarrow \frac{7}{13} R_1(\text{old})$

$R_2(\text{New}) \rightarrow R_2(\text{old}) - \frac{1}{7} R_1(\text{New})$

		c_j	0	0	0	0	
co-eff^n of Basic Variables	Variables in Basis	Sol^n Values	x_1	x_2	S_1	S_2	Min^m ratio
0	x_1	$21/13$	1	0	$-7/13$	$1/13$	
0	x_2	$\frac{10}{13}$	0	1	$\frac{1}{13}$	$-2/13$	
$Z^* = 0$		z_j	0	0	0	0	
		$c_j - z_j$	0	0	0	0	

Phase – II

		c_j	-1	-1	0	0	
co-eff^n of Basic Variables	Variables in Basis	Sol^n values	x_1	x_2	S_1	S_2	Min^m ratio
-1	x_1	$21/13$	1	0	$-7/13$	$1/13$	
-1	x_2	$\frac{10}{13}$	0	1	$1/13$	$\frac{-2}{13}$	
$Z^* = -\frac{31}{13}$		z_j	-1	-1	$6/13$	$1/13$	
		$c_j - z_j$	0	0	$-6/13$	$-1/13$	

Hence, $x_1 = \frac{21}{13}$, $x_2 = \frac{10}{13}$

and the optimum Sol^n $Z^* = -\frac{31}{13}$ Ans

(1) Reduce the following LPP into standard form:-

$$\text{Min } z = x_1 + 2x_2 + 3x_3$$

Subject to

$$2x_1 + 3x_2 + 3x_3 \leq -4$$

$$3x_1 + 5x_2 + 2x_3 \leq 7$$

and $x_1, x_2 \geq 0$, x_3 is unrestricted sign.

Sol^n:- → Standard Form

$$\text{Max } z^* = -x_1 - 2x_2 - 3x_3$$

Subject to $-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$

$$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$$

and $x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$

(2) Reduce in Standard

Minimize $z = 2x_1 + x_2 + 4x_3$

Subject to $-2x_1 + 4x_2 \leq 4$

$$x_1 + 2x_2 + x_3 \geq 5$$

$$2x_1 + 3x_3 \leq 2$$

and $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

Sol^n:- Standard form

$$\text{Max } z^* = -2x_1 - x_2 - 4x_3$$

S.t $-2x_1 + 4x_2 + s_1 = 4$

$$x_1 + 2x_2 + (x_3' - x_3'') - s_2 = 5$$

$$2x_1 + 3(x_3' - x_3'') + s_3 = 2$$

and $x_1, x_2, x_3', x_3'', s_1, s_2, s_3 \geq 0$ Ans

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(3.) $\text{Min}(Z) = 2x_1 + x_2 + 4x_3$

s.t $-2x_1 + 4x_2 \leq -4$

$x_1 + 2x_2 + x_3 \geq 3$

$2x_1 + 3x_2 + 4x_3 \leq 2$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign

Sol^n:- → Standard form

$\text{Max}(Z) = -2x_1 - x_2 - 4x_3$

s.t $2x_1 - 4x_2 - S_1 = 4$

$x_1 + 2x_2 + (x_3' - x_3'') - S_2 = 3$

$2x_1 + 3x_2 + 4(x_3' - x_3'') + S_3 = 2$

and $x_1, x_2, x_3', x_3'', S_1, S_2, S_3 \geq 0$ Ans